Prime Property

Lemma (Prime property):
$\forall p \in \mathbb{P}\ \forall x,y \in \mathbb{Z}\ (p \mid xy \to p \mid x \vee p \mid y)$
Proof: Assume $p \in \mathbb{P}$, $x,y \in \mathbb{Z}$, and $p \mid xy$.
Either $p \mid x$ or $p \nmid x$.
If $p \mid x$ then we're done.
If $p \nmid x$ then $\gcd(p,x) = 1$.
By Bézout's Identity, $\exists a,b \in \mathbb{Z}\ (1 = ax + bp)$
Now $y = y \cdot 1 = y(ax+bp) = axy + byp$
$p \mid xy$ and $p \mid p$ so $p \mid (axy + byp)$ and thus $p \mid y$.
In either case, $p \mid x \vee p \mid y$, as needed.