Greatest Common Divisors Via Prime Factorizations

Claim: Let $p_1, p_2, p_3, \dots$ be the primes in ascending order. Given $x, y \in \mathbb{N}$ with prime factorizations $x = \prod_{j=1}^{\infty} p_j^{a_j}$ and $y = \prod_{j=1}^{\infty} p_j^{b_j}$ then

$$\gcd(x,y) = \prod_{j=1}^{\infty} p_j^{\min(a_j, b_j)}$$

Example: If $x = 140$ and $y = 168$ then $x = 2^2 3^0 5^1 7^1$ and $y = 2^3 3^1 5^0 7^1$ so $\gcd(x,y) = 2^{\min\{2,3\}} 3^{\min\{0,1\}} 5^{\min\{1,0\}} 7^{\min\{1,1\}} = 2^2 \cdot 7 = 28$

Proof:

$$Div(x) = \left\{ \prod_{j=1}^{\infty} p_j^{c_j} : \forall j \in \mathbb{N}\, (c_j \le a_j) \right\}$$

$$Div(y) = \left\{ \prod_{j=1}^{\infty} p_j^{c_j} : \forall j \in \mathbb{N}\, (c_j \le b_j) \right\}$$

$$Div(x) \cap Div(y) = \left\{ \prod_{j=1}^{\infty} p_j^{c_j} : \forall j \in \mathbb{N}\, (c_j \le \min\{a_j, b_j\}) \right\} = Div\left(\prod_{j=1}^{\infty} p_j^{\min\{a_j, b_j\}} \right)$$

$$\gcd(x,y) = \prod_{j=1}^{\infty} p_j^{\min\{a_j, b_j\}}$$

Note: $Div(x) \cap Div(y) = Div(\gcd(x,y))$