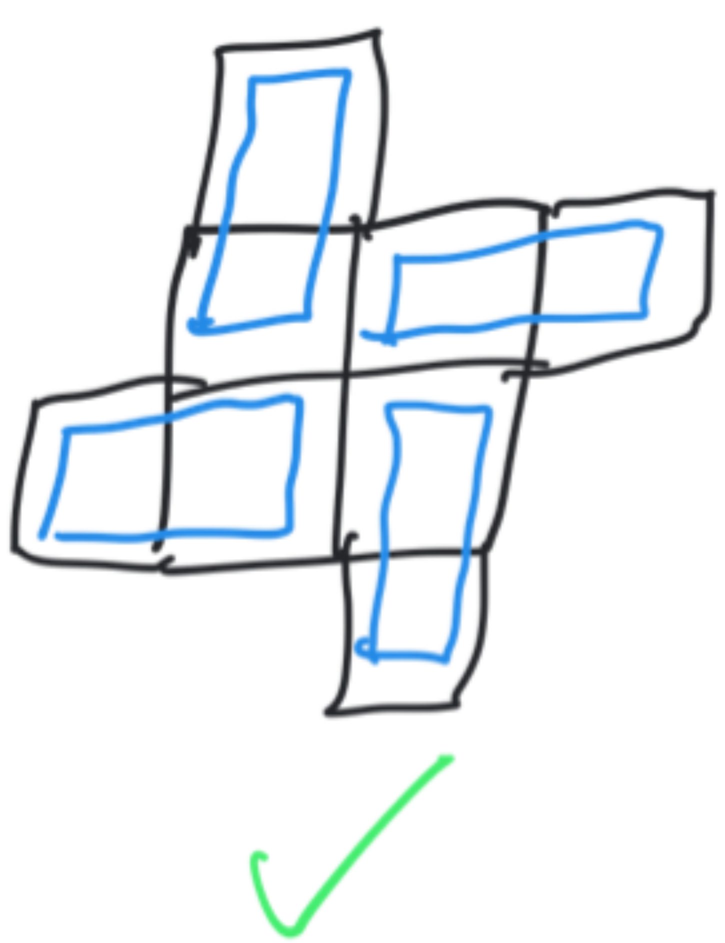
Invariants

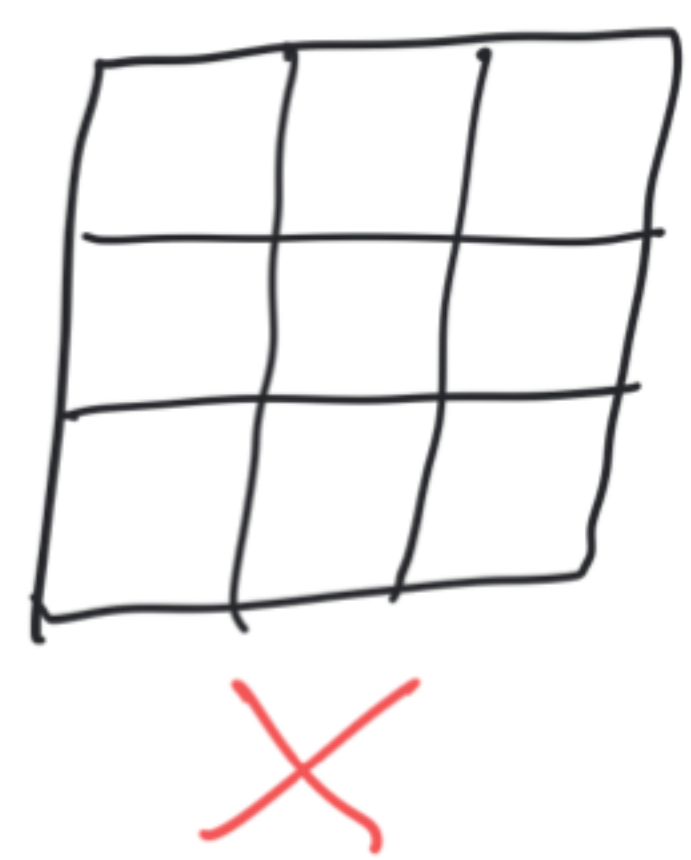
When analyzing a process, it can be very useful to find invariants which remain unchanged.

Example: when running Euclid's algorithm, $Div(a) \cap Div(b)$ and $span\{a,b\}$ are invariants.

Domino Tilings: Given a shape, is it possible to cover it with non-overlapping 2×1 dominos?

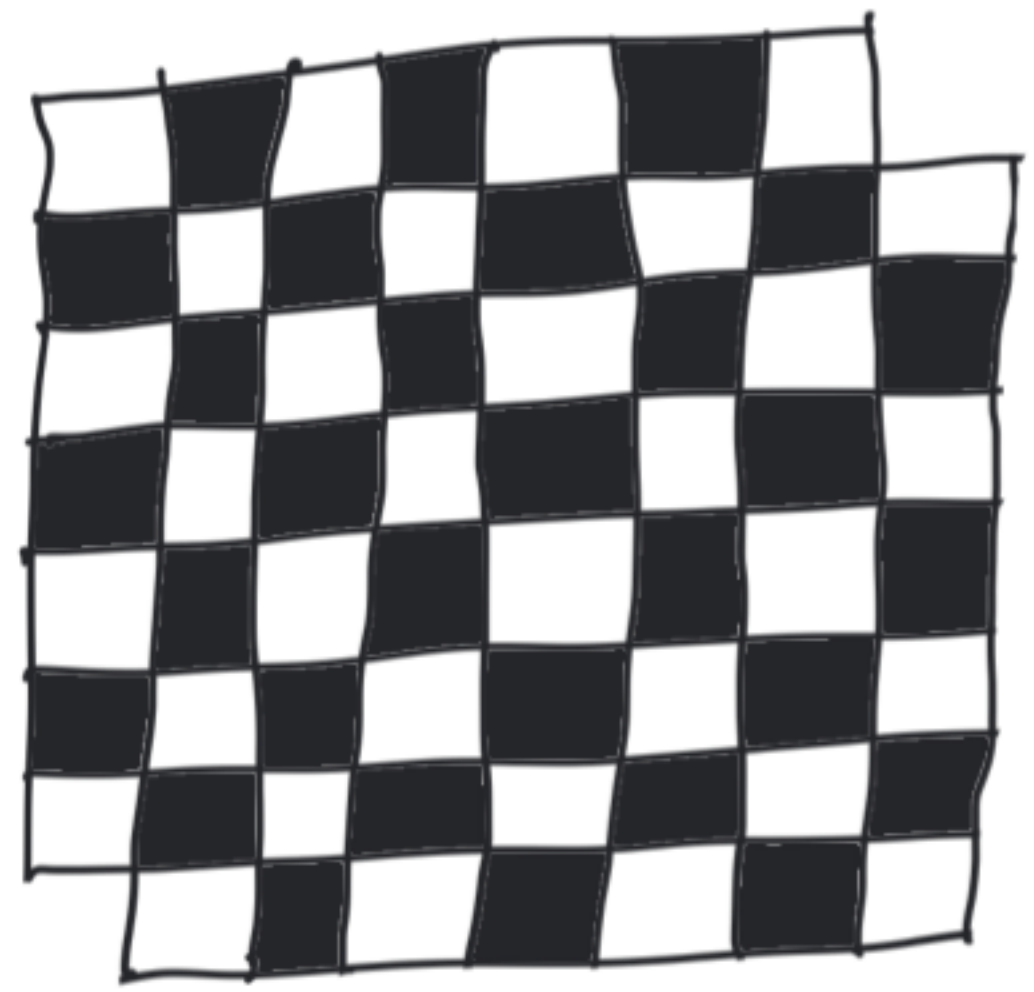
Examples:





There are an odd number of squares.

Invariants



Mutilated Chessboard

Q: Can the mutilated chessboard be tiled?

Theorem: There is no domino tiling of the mutilated chessboard.

Proof: Observe that as we place dominos,
# of uncovered white squares − # of uncovered black squares
is invariant.
Thus, any shape which can be tiled must have an equal # of white and black squares.
However, the mutilated chessboard has 32 white squares and 30 black squares.