Perfect Numbers

Definition: A *perfect number* is a natural number n such that the sum of the positive divisors of n (excluding n) is equal to n. Equivalently, the sum of the positive divisors of n (including n) is $2n$.

Examples: 6 is a perfect number as $1+2+3=6$.

28 is a perfect number as $1+2+4+7+14=28$

$496 = 2^4 \cdot 31$ is a perfect number as the sum of the positive divisors of 496 is $(1+2+4+8+16)\cdot(1+31) = 31\cdot 32 = 31\cdot 2^5 = 2\cdot 496$

Perfect Numbers

Pattern: $6 = 2 \cdot 3 = 2^1(2^2-1)$

$28 = 4 \cdot 7 = 2^2(2^3-1)$

$496 = 16 \cdot 31 = 2^4(2^5-1)$

Theorem: The even perfect numbers are $\{2^k(2^{k+1}-1) : k \in \mathbb{N} \wedge 2^{k+1}-1 \in \mathbb{P}\}$

Note: Primes of the form 2^k-1 are called Mersenne primes.

Open question 1: Are there any odd perfect numbers?

Open question 2: Are there infinitely many Mersenne primes?