Sets

Definition: A set is an unordered collection of objects. To describe a set, we use $\{$ and $\}$ brackets and we put the objects (or a description of the objects) between the brackets.

Example: $S = \{$Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday$\}$ is the set of days of the week.

Definition: If an object x is contained in a set S then we write that $x \in S$ ($\in$ = is in). Otherwise, we write that $x \notin S$ ($\notin$ = is not in).

Example: Wednesday $\in S$ but Halloween $\notin S$.

Important note: Sets ignore duplicates and ordering so $\{3,3,2,1,1,1\}$ is the same as $\{1,2,3\}$.

To keep track of the number of times each object appears, we use multi-sets.

Important Sets

1. $\emptyset = \{\}$ is the empty set.
2. $\mathbb{N} = \{1, 2, 3, \ldots\}$ is the set of natural numbers.
3. $[n] = \{x \in \mathbb{N} : 1 \leq x \leq n\}$ is the set of natural numbers between 1 and n.
4. $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of integers.
5. $\mathbb{Q} = \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$ is the set of rational numbers.
6. $\mathbb{R}$ is the set of real numbers.
7. $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$ is the closed interval between a and b.
   $(a, b) = \{x \in \mathbb{R} : a < x < b\}$ is the open interval between a and b.
8. $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$ (where $i = \sqrt{-1}$) is the set of complex numbers.
9. $\mathbb{P} = \{p \in \mathbb{N} : p > 1$ and p and 1 are the only positive divisors of $p\}$ is the set of prime numbers.

Describing Sets

1. If the set only has a few objects/elements, we can just list these objects/elements.

   Example: $[4] = \{1, 2, 3, 4\}$

2. We can use ... to indicate a continuing pattern.

   Note: This should only be used for simple patterns and it should be accompanied by an English description of the elements of the set.

   Examples: $\{0, 1, 4, 9, 16, \ldots\}$ is the set of square numbers.

   $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$ is the set of even integers.

3. Set-builder Notation:

   $\{ \qquad : \qquad \}$

   elements of the set $\overset{\text{or}}{|}$ conditions on the elements of the set

   Example: $S = \{n^2 : n \in \mathbb{Z}\}$ is the set of square numbers.

Set Builder Notation Examples

Q: How can we write the set S of numbers which are the sum of two squares?

A: $S = \{a^2+b^2 : a, b \in \mathbb{Z}\}$

Remark: This sequence of numbers $0, 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, 20, \ldots$ is beautiful.

Challenge: Can you figure out how to determine if a natural number $n \in S$ based on its prime factorization?

Q: How can we write the set T of natural numbers n such that $2n+1$ is prime?

A: $T = \{n : n \in \mathbb{N} \wedge 2n+1 \in \mathbb{P}\}$

Note: If the elements are described by a single letter, we can shift the domain to the left side.

Example: $T = \{n \in \mathbb{N} : 2n+1 \in \mathbb{P}\}$