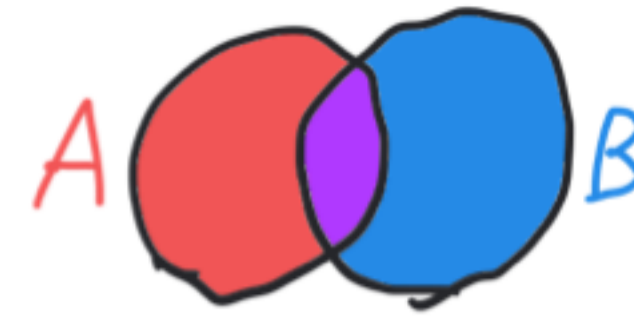


Operations on Sets



1. Union: $A \cup B = \{x: x \in A \vee x \in B\}$



2. Intersection: $A \cap B = \{x: x \in A \wedge x \in B\}$



3. Set difference: $A \setminus B = \{x: x \in A \wedge x \notin B\}$



4. Complement: If we are living in some universe U of objects, $\bar{A} = \{x \in U: x \notin A\}$



5. Cartesian product: $A \times B = \{(a, b): a \in A, b \in B\}$



6. Cardinality: The **cardinality** $|A|$ of a finite set A is the # of elements in A .
 Remark: The cardinality of infinite sets is a very interesting topic.

Definition: We say that $A \subseteq B$ if whenever $x \in A$, $x \in B$ as well.
 We say that $A = B$ if $A \subseteq B$ and $B \subseteq A$, i.e. $x \in A \leftrightarrow x \in B$.
 If $A \subseteq B$ but $A \neq B$, we write that $A \subset B$. This can also be written as $A \subsetneq B$.

Examples

Let $A = \{1, 2, 5\}$ and let $B = \{2, 4\}$

$$A \cup B = \{1, 2, 4, 5\}$$

$$A \cap B = \{2\}$$

$$A \setminus B = \{1, 5\}$$

$$A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (5, 2), (5, 4)\}$$

$$|A| = 3, |B| = 2, |A \cup B| = 4, |A \cap B| = 1, |A \setminus B| = 2, \text{ and } |A \times B| = 6$$

Complement example: If $U = \mathbb{Z}$ and $E = \{2n : n \in \mathbb{Z}\}$ then $\overline{E} = \{2n+1 : n \in \mathbb{Z}\}$

Subset examples:

$$A \cap B = \{2\} \subseteq \{1, 2, 5\} = A$$

$$A = \{1, 2, 5\} \subseteq \{1, 2, 4, 5\} = A \cup B$$

Note: There's a distinction between an element x and the set $\{x\}$ which only contains x .

Correct:

$$2 \in A$$

$$\{2\} \subseteq A$$

Incorrect:

$$2 \subseteq A$$

$$\{2\} \in A$$