Rules for Negation

Reasons for negating a proposition P:

1. Negating P tells us how to prove P is false.
2. One way to prove P is a proof by contradiction where we assume $\neg P$ and derive $\perp$ (FALSE).

---

1. De Morgan's Laws

$$\neg(P \vee Q) \leftrightarrow \neg P \wedge \neg Q$$

$$\neg(P \wedge Q) \longleftrightarrow \neg P \vee \neg Q$$

2. $\neg(P \rightarrow Q) \longleftrightarrow P \wedge \neg Q$
3. The negation of $\forall x(P(x))$ is $\exists x(\neg P(x))$
4. The negation of $\exists x(P(x))$ is $\forall x(\neg P(x))$

Negation Examples

$P: A \to (B \wedge C)$

$\neg P: A \wedge \neg(B \wedge C) \Longleftrightarrow A \wedge (\neg B \vee \neg C)$

$Q: \exists n \in \mathbb{N}\,(n > 4 \wedge 2^{(2^n)} + 1 \in \mathbb{P})$

$\neg Q: \forall n \in \mathbb{N}\,(n \leq 4 \vee 2^{(2^n)} + 1 \notin \mathbb{P})$

Goldbach Conjecture:

$G: \forall n \in \mathbb{N}\,(n \leq 1 \vee \exists p, q \in \mathbb{P}\,(2n = p + q))$

$\neg G: \exists n \in \mathbb{N}\,(\neg(n \leq 1 \vee \exists p, q \in \mathbb{P}\,(2n = p + q)))$

$\neg G: \exists n \in \mathbb{N}\,(n > 1 \wedge \neg(\exists p, q \in \mathbb{P}\,(2n = p + q)))$

$\neg G: \exists n \in \mathbb{N}\,(n > 1 \wedge \forall p, q \in \mathbb{P}\,(2n \neq p + q))$