

Truth Tables

A **truth table** is a table which says whether a proposition is true or false based on whether the atomic propositions it is composed of are true or false.

Examples:

P	$\neg P$
F	T
T	F

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

P	Q	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T

P	Q	$P \leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

Tautologies

A **tautology** is a proposition which is true simply because of its structure.

To quote XKCD 703, "The first rule of tautology club is the first rule of tautology club."

A proposition is a tautology if and only if every entry in its truth table is true.

Example 1: $P \vee \neg P$ is a tautology.

P	$\neg P$	$P \vee \neg P$
F	T	T
T	F	T

Example 2: $Q \rightarrow (P \rightarrow Q)$ is a tautology.

P	Q	$P \rightarrow Q$	$Q \rightarrow (P \rightarrow Q)$
F	F	T	T
F	T	T	T
T	F	F	T
T	T	T	T

Example 3: $P \vee Q$ is not a tautology.

P	Q	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

Checking Equivalence Using Truth Tables

Two propositions are equivalent if and only if they have the same truth tables.

Example 1: Are $P \rightarrow Q$ and $\neg Q \rightarrow \neg P$ equivalent?

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T

P	Q	$\neg Q$	$\neg P$	$\neg Q \rightarrow \neg P$
F	F	T	T	T
F	T	F	T	T
T	F	T	F	F
T	T	F	F	T

YES! Note: $\neg Q \rightarrow \neg P$ is called the *contrapositive* of $P \rightarrow Q$.

Example 2: Are $\neg(P \wedge Q)$ and $\neg P \vee \neg Q$ equivalent?

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	F

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	T
T	T	F	F	F

YES! This is one of *De Morgan's Laws*.

Checking Equivalence Using Truth Tables

Example 3: Are $P \rightarrow Q$ and $Q \rightarrow P$ equivalent?

P	Q	$P \rightarrow Q$	$Q \rightarrow P$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

No.

Note: $Q \rightarrow P$ is called the *converse* of $P \rightarrow Q$.

Example 4: Are $(P \wedge Q) \vee R$ and $P \wedge (Q \vee R)$ equivalent?

P	Q	R	$P \wedge Q$	$(P \wedge Q) \vee R$
F	F	F	F	F
F	F	T	F	T
F	T	F	F	F
F	T	T	F	T
T	F	F	F	F
T	F	T	F	T
T	T	F	T	T
T	T	T	T	T

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$
F	F	F	F	F
F	F	T	T	F
F	T	F	T	F
F	T	T	T	F
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

No.

Drawback of Truth Tables

Drawback: If there are n atomic propositions then the truth table will have 2^n rows.

This is only practical for very small n .

Next lecture: Deduction rules