

Propositions

Definition: A **proposition** is a statement which is either true or false (though you may not know whether it is true or false).

An **atomic proposition** is a proposition which cannot be broken into simpler parts.

Examples:

A: Pigs can sink.

B: Pigs can fly.

C: Springfield is the capital of Illinois.

D: It will be sunny tomorrow.

} atomic propositions

Example: Pigs can sink and pigs can fly is not an atomic proposition.

Note: In mathematics, a proposition is a statement which is almost self-evident and thus needs little if any proof.

Logical Connectives

To build propositions from atomic propositions, we can use **logical connectives**.

Logical connectives:

\neg (NOT)

\wedge (AND)

\vee (OR)

\rightarrow (IMPLIES)

\leftrightarrow (EQUIVALENCE)

$\neg P$ is true if and only if P is false.

$P \wedge Q$ is true if and only if P and Q are both true.

$P \vee Q$ is true if P is true or Q is true (or both).

$P \rightarrow Q$ is true if P is false or Q is true.

$P \leftrightarrow Q$ is true if and only if P and Q are both false or P and Q are both true.

Logical Connective Examples

A: Pigs can oink. T
B: Pigs can fly. F
C: Springfield is the capital of Illinois. T
D: It will be sunny tomorrow. ?

Examples:

$\neg B$: Pigs cannot fly. T
 $A \wedge B$: Pigs can oink and pigs can fly. F
 $A \vee C$: Pigs can oink or Springfield is the capital of Illinois. T
 $B \vee D$: Pigs can fly or it will be sunny tomorrow. ?
 $A \rightarrow B$: If pigs can oink then pigs can fly. F
 $B \rightarrow A$: If pigs can fly then pigs can oink. T

Order of Operations

1. \neg
2. \wedge
3. \vee
4. \rightarrow
5. \leftrightarrow

Examples:

$\neg A \wedge B$ says that A is false and B is true

$\neg(A \wedge B)$ says that A and B are not both true.

$A \rightarrow B \vee C$ says that if A is true then either B is true or C is true (or both).

$(A \rightarrow B) \vee C$ says that either A implies B or C is true.