

Examples of Number Theory Proofs

What you should take away from
Propositional Logic: Structure of proofs

Example: Prove that for all integers x , $x(x+1)$ is even.

Proof:

Let x be an integer.

x is either even or odd.

If x is even then $x(x+1)$ is even.

If x is odd then $x+1$ is even so $x(x+1)$ is even.

Thus, $x(x+1)$ is even.

A : x is even B : x is odd C : $x(x+1)$ is even

$A \vee B$

$A \rightarrow C$

$B \rightarrow C$

$A \vee B, A \rightarrow C, B \rightarrow C \vdash C$

Examples of Number Theory Proofs

Example: Show that $3^5 - 3$ is divisible by 5.

Direct proof:

$$3 \cdot 3 = 9 \quad 3 \cdot 9 = 27 \quad 3 \cdot 27 = 81 \quad 3 \cdot 81 = 243 \quad 3^5 = 243$$

$$3^5 - 3 = 240 = 48 \cdot 5 \quad \text{is divisible by 5.}$$

Fermat's Little Theorem: For all integers x and p , if p is prime then $x^p - x$ is divisible by p .

Proof using Fermat's Little Theorem:

By Fermat's Little Theorem, if p is prime then

$3^p - 3$ is divisible by p .

5 is prime, so $3^5 - 3$ is divisible by 5.

A: 5 is prime B: $3^5 - 3$ is divisible by 5.

A

$A \rightarrow B$

$A, A \rightarrow B \vdash B$