

## List of Additional Deduction Rules

1.  $\wedge$  and  $\vee$  are commutative  
 $A \wedge B \leftrightarrow B \wedge A$   
 $A \vee B \leftrightarrow B \vee A$
2.  $\wedge$  and  $\vee$  are associative  
 $A \wedge (B \wedge C) \leftrightarrow (A \wedge B) \wedge C$   
 $A \vee (B \vee C) \leftrightarrow (A \vee B) \vee C$
3.  $\rightarrow$  is transitive  
 $A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$
4. De Morgan's Laws  
 $\neg(A \vee B) \leftrightarrow \neg A \wedge \neg B$   
 $\neg(A \wedge B) \leftrightarrow \neg A \vee \neg B$
5.  $\ast A \rightarrow B \leftrightarrow \neg A \vee B$

6.  $A \vee B, \neg A \vdash B$

7. Resolution rule:

$$A \vee B, \neg B \vee C \vdash A \vee C$$

8. Distributive Laws:

$$A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$$

$$A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$$

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$\ast$  Law of the Excluded Middle:

$$A \vee \neg A$$

$\ast$  Principle of Double Negation:

$$\neg\neg A \rightarrow A$$

$\ast$  Law of Contraposition:

$$A \rightarrow B \leftrightarrow \neg B \rightarrow \neg A$$

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$\ast$  Showing these deduction rules requires the Law of the Excluded Middle, the Principle of Double Negation, or the Law of Contraposition.



## Other General Rules

1. If  $A \leftrightarrow B$ , we can replace  $A$  with  $B$  and vice versa.

Example:  $A \leftrightarrow B, A \vee C \vdash B \vee C$

2. Replacing  $\vdash$  with  $\rightarrow$  (and the commas before  $\vdash$  with  $\wedge$ ) gives a tautology.

Example 1:  $A \vdash A \vee B$

$A \rightarrow A \vee B$  is a tautology.

Example 2:  $A, A \rightarrow B \vdash B$

$A \wedge (A \rightarrow B) \rightarrow B$  is a tautology.