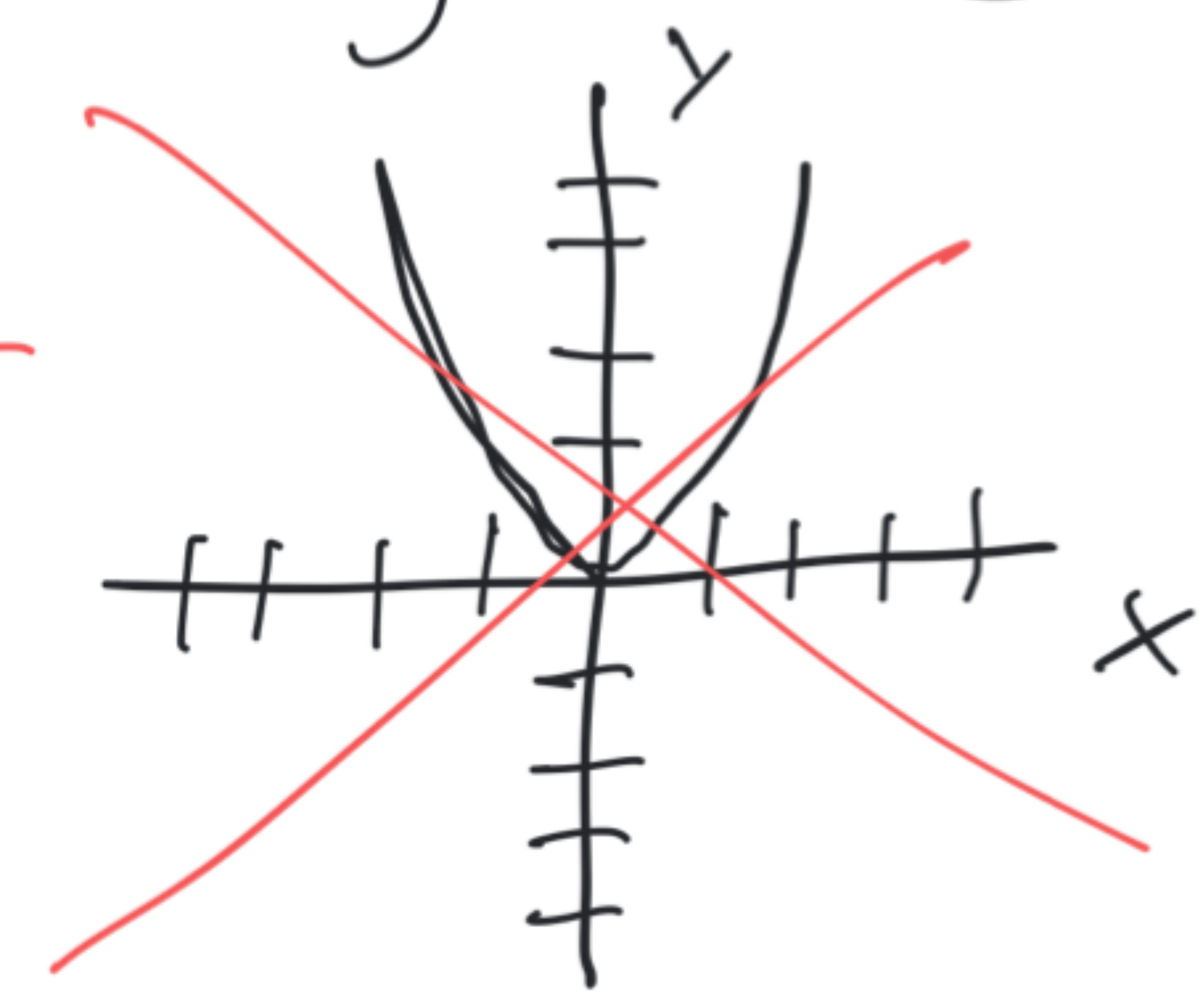
Introduction to Graph Theory

Graph Theory is

1. Extremely useful
2. Elementary
3. Very important for Theory of Algorithms

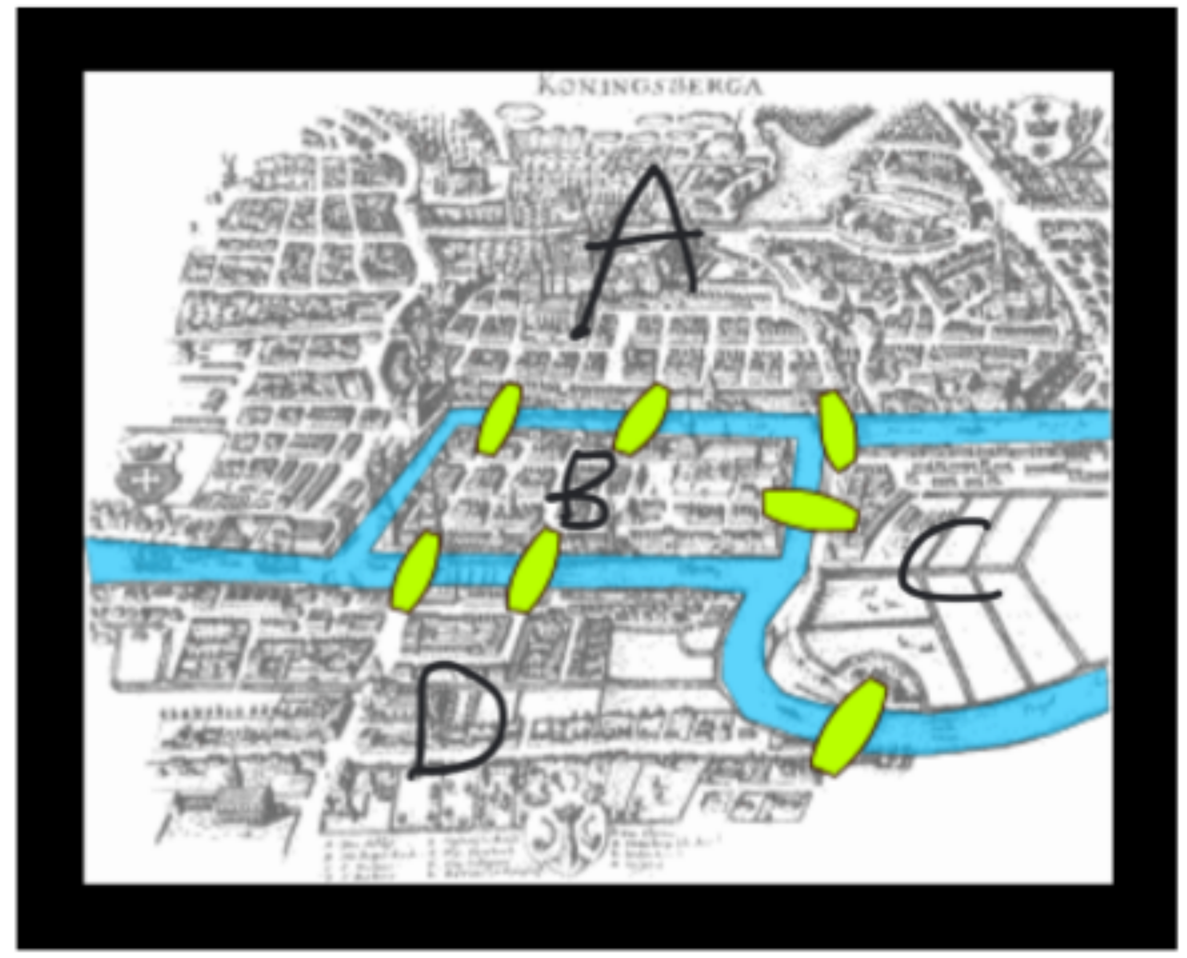
Note: Graph theory is NOT about these kinds of graphs:



$y=x^2$

In graph theory, graphs consist of vertices (which generally correspond to people, places, or things) and edges between the vertices (which generally correspond to connections between the people, places, or things.)

Examples of Graphs



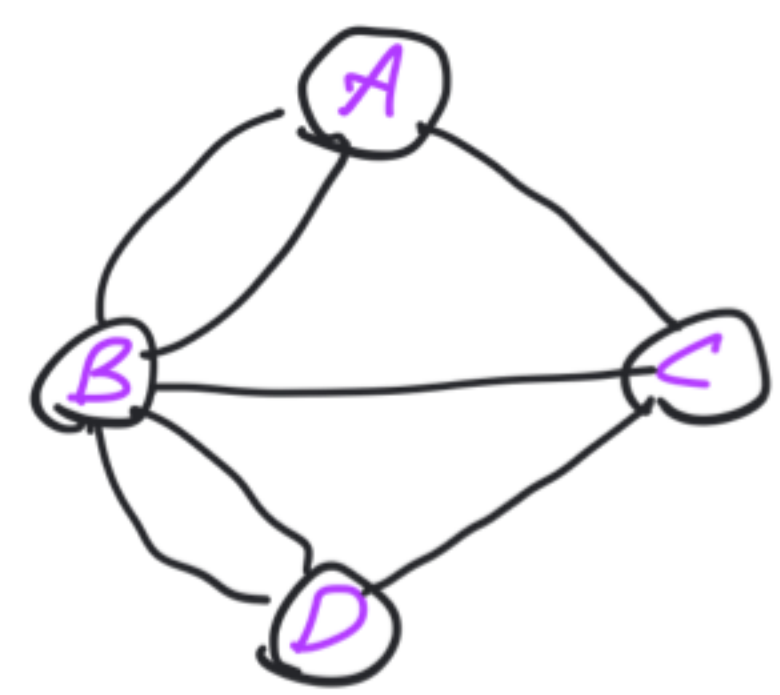
Bridges of Königsberg in Euler's time (Image is from Wikipedia)

We can describe how the bridges connect Königsberg as follows:

Vertices: Parts of Königsberg

Edges: Bridges

Graph:



Examples of Graphs

People: Alice, Bob, Charlie, Diana, Emily, and Frank

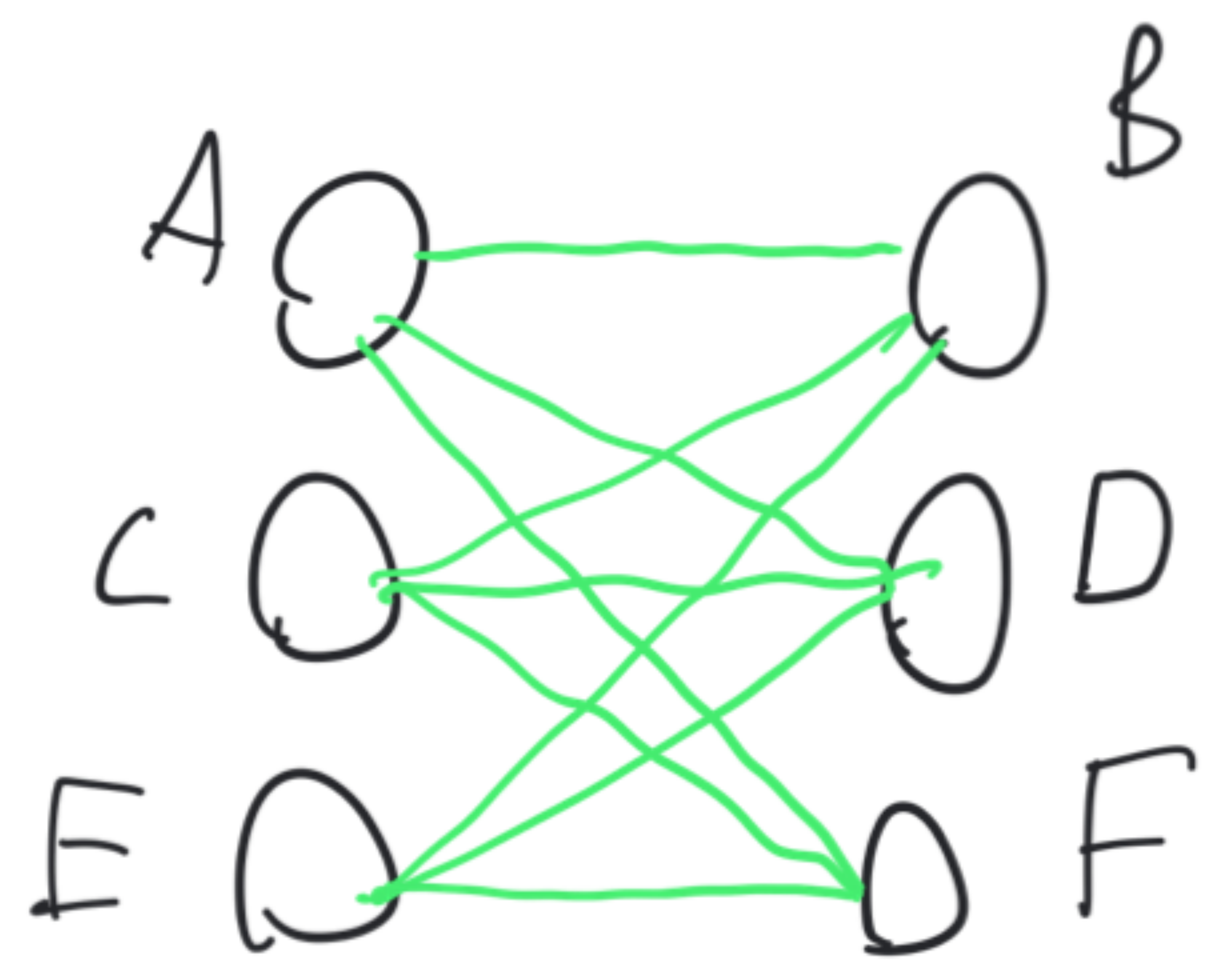
	A	B	C	D	E	F
A	—	✓	✗	✓	✗	✓
B	✓	—	✓	✗	✓	✗
C	✗	✓	—	✓	✗	✓
D	✓	✗	✓	—	✓	✗
E	✗	✓	✗	✓	—	✓
F	✓	✗	✓	✗	✓	—

Friendship Table

Graph:

Vertices: People

Edges: Friendships



Graph Definition

Definition: An undirected graph G consists of a set of vertices $V(G)$ and a set of edges $E(G)$ where each edge $e \in E(G)$ is an unordered pair of vertices in $V(G)$ (i.e. $e = \{u,v\}$ for some $u,v \in V(G)$)

We draw each vertex $v \in V(G)$ as a circle
We draw each edge $e = \{u,v\} \in E(G)$ as a line between u and v.

Example: If $V(G) = \{v_1, v_2, v_3\}$ and $E(G) = \{\{v_1,v_2\},\{v_1,v_3\},\{v_2,v_3\}\}$

G

Definition: A directed graph G is defined in the same way except that each edge $e \in E(G)$ is an ordered pair of vertices (i.e. $e = (u,v)$ for some $u,v \in V(G)$) and is drawn as a line from u to v with an arrow pointing from u to v.

Example: If $V(G) = \{v_1, v_2, v_3\}$ and $E(G) = \{(v_1,v_2),(v_2,v_3),(v_3,v_1)\}$

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