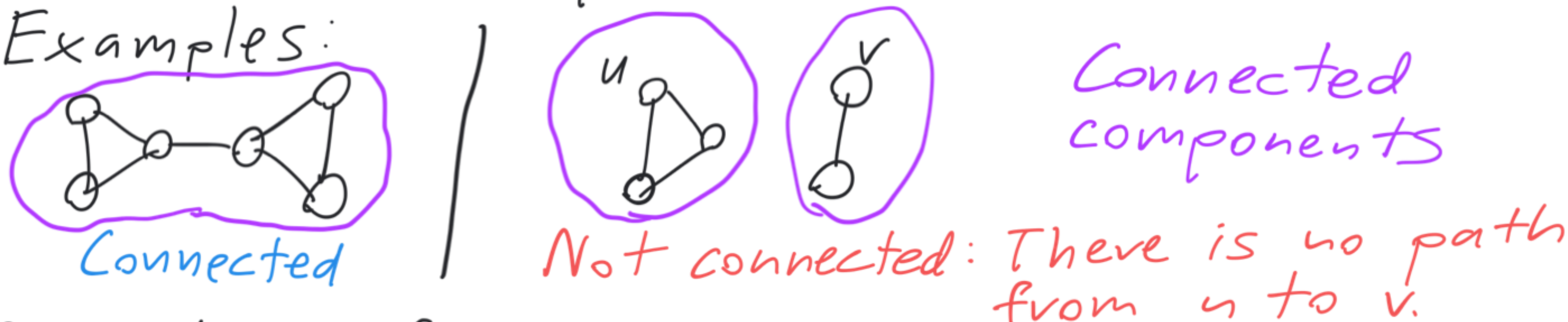
Connectivity and Connected Components

Definition: We say that an <u>undirected</u> graph G is connected if for all vertices $u, v \in V(G)$ there exists a path from u to v in G.

Examples:



Definition: Given an undirected graph G, a connected component of G is a set of vertices $U \subseteq V(G)$ such that:

1) $\forall u, u' \in U$ (there is a path from u to u' in G)

2) $\forall u \in U \; \forall v \notin U$ (there is no path from u to v in G)

Proposition: Every <u>undirected</u> graph G can be partitioned into connected components.

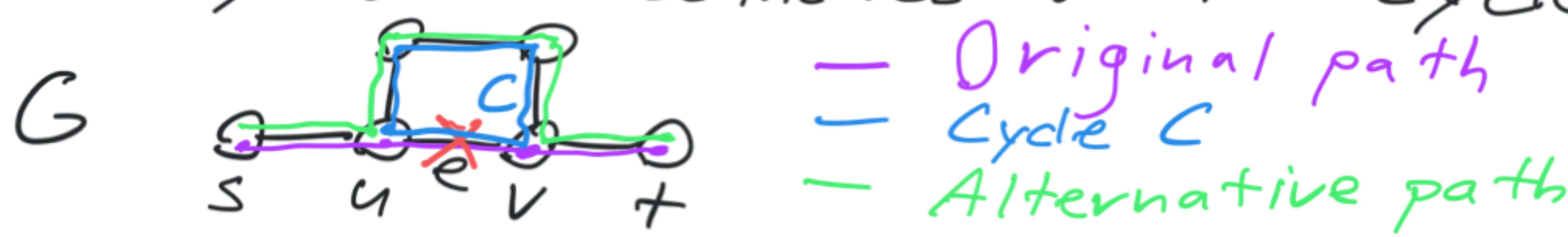
Connectivity and Connected Components

Q: If we delete an edge $e = \{u, v\}$ from an undirected graph G, how does this affect the connected components of G?

Claim: If e is contained in a cycle C, deleting e does not affect the connected components of G.

Idea: Instead of using e to go from a vertex s to another vertex t, we can use the rest of the cycle C.

Example:



Connectivity and Connected Components

Q: If we delete an edge $e = \{u, v\}$ from an undirected graph G, how does this affect the connected components of G?

Claim: If $e = \{u, v\}$ is not contained in a cycle then deleting e splits the connected component containing e into two connected components

$U = \{u' \in V(G) :$ there is a path from u to u' in G which does not contain $e\}$

$V = \{v' \in V(G) :$ there is a path from v to v' in G which does not contain $e\}$

Observations:

1) $U \cup V$ is the original connected component containing e.
2) After deleting e, U and V are connected components
3) There is no path from u to v which does not contain e as otherwise e would be contained in a cycle. Thus, $v \notin U$ so U and V are separate connected components.

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Claim: If $e = \{u, v\}$ is not contained in a cycle then deleting e splits the connected component containing e into two connected components

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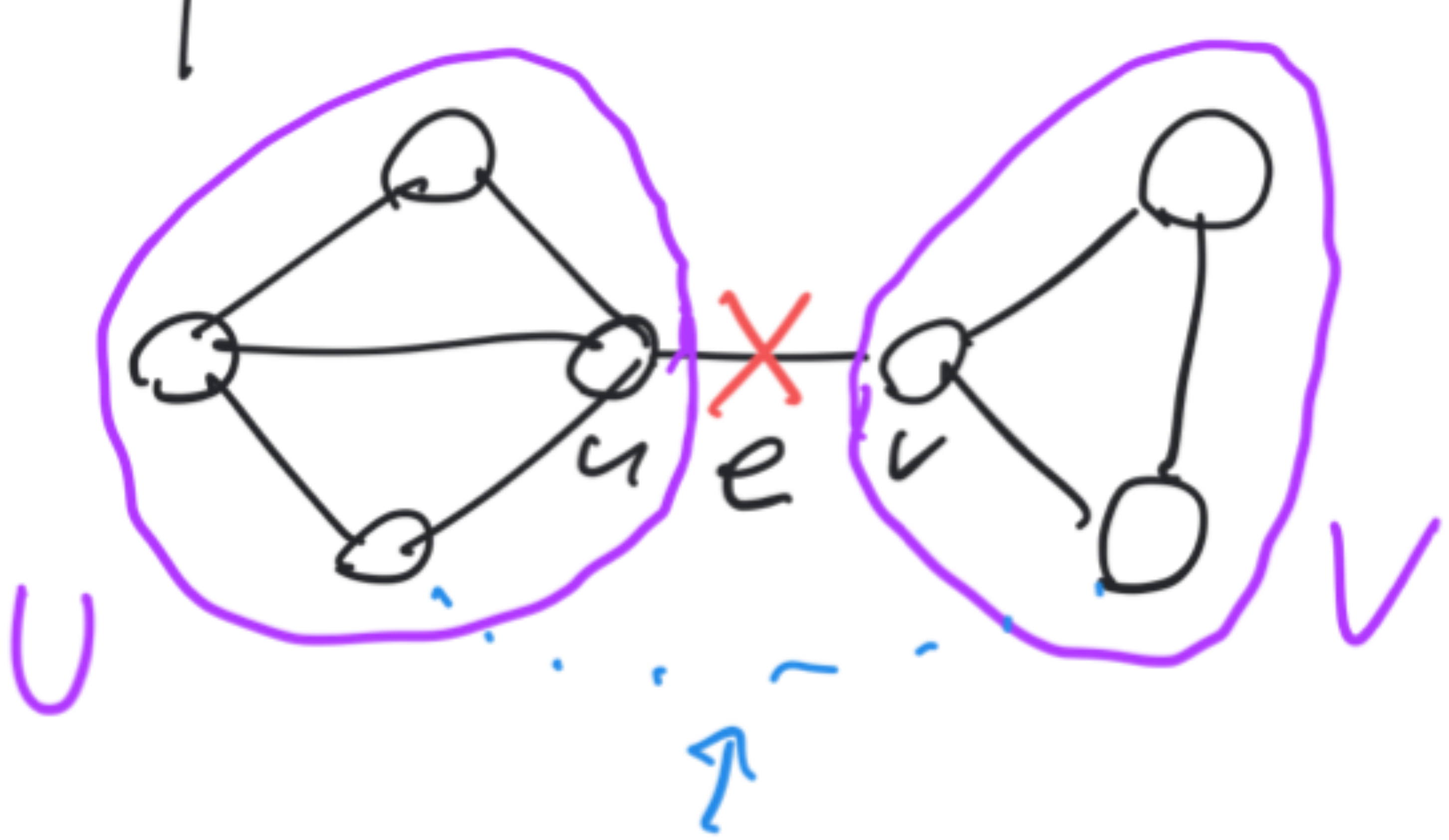

Note: If we had a path from U to V which did not contain e, this would create a cycle containining e.