Solving the Fibonacci Recurrence Relation

Fibonacci numbers recurrence relation: $F_0=0,\ F_1=1,\ F_n=F_{n-1}+F_{n-2}$

Trying $F_n = x^n$

$x^n = x^{n-1} + x^{n-2}$

Dividing everything by x^{n-2}

$x^2 = x + 1$

$x^2 - x - 1 = 0$ $\qquad x = \dfrac{1 \pm \sqrt{1+4}}{2} = \dfrac{1 \pm \sqrt{5}}{2}$

General solution: $F_n = c_1\left(\dfrac{1+\sqrt{5}}{2}\right)^n + c_2\left(\dfrac{1-\sqrt{5}}{2}\right)^n$

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$$F_0 = c_1 + c_2 = 0$$

$$c_2 = -c_1$$

$$F_1 = \frac{1}{2}c_1 + \frac{\sqrt{5}}{2}c_1 + \frac{1}{2}c_2 - \frac{\sqrt{5}}{2}c_2$$

$$= \cancel{\frac{1}{2}c_1} + \frac{\sqrt{5}}{2}c_1 - \cancel{\frac{1}{2}c_1} + \frac{\sqrt{5}}{2}c_1 = \sqrt{5}c_1 = 1$$

$$c_1 = \frac{1}{\sqrt{5}} \qquad c_2 = -\frac{1}{\sqrt{5}}$$

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$