

Introduction to Recurrence Relations

What is a recurrence relation?

Let's say we have a sequence of numbers a_1, a_2, a_3, \dots which we want to describe.

One way: Describe each number as a function of n .

Examples:

$$a_n = n^2$$

$$b_n = 2^n$$

n	1	2	3	4	5	6	7	8
a_n	1	4	9	16	25	36	49	64
b_n	2	4	8	16	32	64	128	256

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Another way: Give the first few number(s) and then give a recurrence relation describing each number in terms of the previous numbers.

Examples:

$$a_1 = 1, a_n = a_{n-1} + 2$$

$$b_1 = 2, b_n = 2b_{n-1} - 1$$

Fibonacci numbers:

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$$

n	1	2	3	4	5	6	7	8
a_n	1	3	5	7	9	11	13	15
b_n	2	3	5	9	17	33	65	129
F_n	1	1	2	3	5	8	13	21

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Q: Given a recurrence relation, can we solve the recurrence relation by describing each number as a function of n ?

Examples:

$$a_1 = 1, a_n = a_{n-1} + 2$$

$$b_1 = 2, b_n = 2b_{n-1} - 1$$

Fibonacci numbers:

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$$

Solutions:

$$a_n = 2n - 1$$

$$b_n = 2^{n-1} + 1$$

Fibonacci numbers: We'll see!

n	1	2	3	4	5	6	7	8
a_n	1	3	5	7	9	11	13	15
b_n	2	3	5	9	17	33	65	129
F_n	1	1	2	3	5	8	13	21

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Some ways to solve recurrence relations:

1. Taking an educated guess and adjusting it as needed.
2. Expanding out the recurrence relation.
3. (More advanced) Generating functions.