Educated Guessing and Checking

Example 1: $a_1=1,\ a_n=2a_{n-1}+3$

Homogeneous part: $a_n = 2a_{n-1}$

Trying $a_n = x^n$, $x^n = 2x^{n-1}$ so $x=2$.

General solution: $c_1 \cdot 2^n$

Single solution: Try $a_n = c$.

$c = 2c+3$ so $c=-3$

One solution: $a_n = -3$

General solution: $a_n = c_1 \cdot 2^n - 3$

Since $a_1=1$, $1 = 2c_1 - 3$ so $c_1 = 2$

Solution: $a_n = 2 \cdot 2^n - 3 = \boxed{2^{n+1} - 3}$

Checking our answer

n	1	2	3	4	5
a_n	1	5	13	29	61
$2^{n+1}-3$	1	5	13	29	61

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Educated Guessing and Checking

Example 2: $b_1=1,\ b_n=3b_{n-1}-1$

Homogeneous part: $b_n=3b_{n-1}$

Trying $b_n=x^n$, $x^n=3x^{n-1}$ so $x=3$

General solution: $c_1\cdot 3^n$

Single solution: Try $b_n=c$

$c=3c-1$ so $c=\frac{1}{2}$

One solution: $b_n=\frac{1}{2}$

General solution: $b_n=c_1\cdot 3^n+\frac{1}{2}$

Since $b_1=1$, $1=3c_1+\frac{1}{2}$ so $c_1=\frac{1}{6}$

Solution: $b_n=\frac{1}{6}\cdot 3^n+\frac{1}{2}=\frac{3^{n-1}+1}{2}$

Checking our answer

n	1	2	3	4	5
b_n	1	2	5	14	41
$\frac{3^{n-1}+1}{2}$	1	2	5	14	41

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