Bayes' Theorem

Bayes' Theorem: $\Pr(A|B) = \dfrac{\Pr(B|A)\Pr(A)}{\Pr(B)}$

Proof: $$\frac{\Pr(B|A)\Pr(A)}{\Pr(B)} = \frac{\frac{\Pr(A\cap B)}{\cancel{\Pr(A)}} \cdot \cancel{\Pr(A)}}{\Pr(B)} = \frac{\Pr(A\cap B)}{\Pr(B)} = \Pr(A|B)$$

Bayes' Theorem is useful for updating a probability estimate based on new information.

Bayes' Theorem Example

Bayes' Theorem: $\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}$

Example: Let's say we've testing for a disease which 1% of the population has and we have a test which is 90% accurate, i.e. if a person has the disease, Pr(test is positive) = 90% and if a person does not have the disease then Pr(test is negative) = 90%.

Q: If we test someone at random and the test comes back positive, what is the probability that they actually have the disease?

A: Person has the disease B: test is positive

$Pr(A) = .01$, $Pr(B|A) = .9$, $Pr(B) = Pr(A)Pr(B|A) + Pr(\neg A)Pr(B|\neg A)$

$= .01 \cdot .9 + .99 \cdot .1$

$= .009 + .099 = .108$

$Pr(A|B) = \frac{.9 \cdot .01}{.108} = \frac{.009}{.108} = \frac{1}{12} \approx 8.33\%$