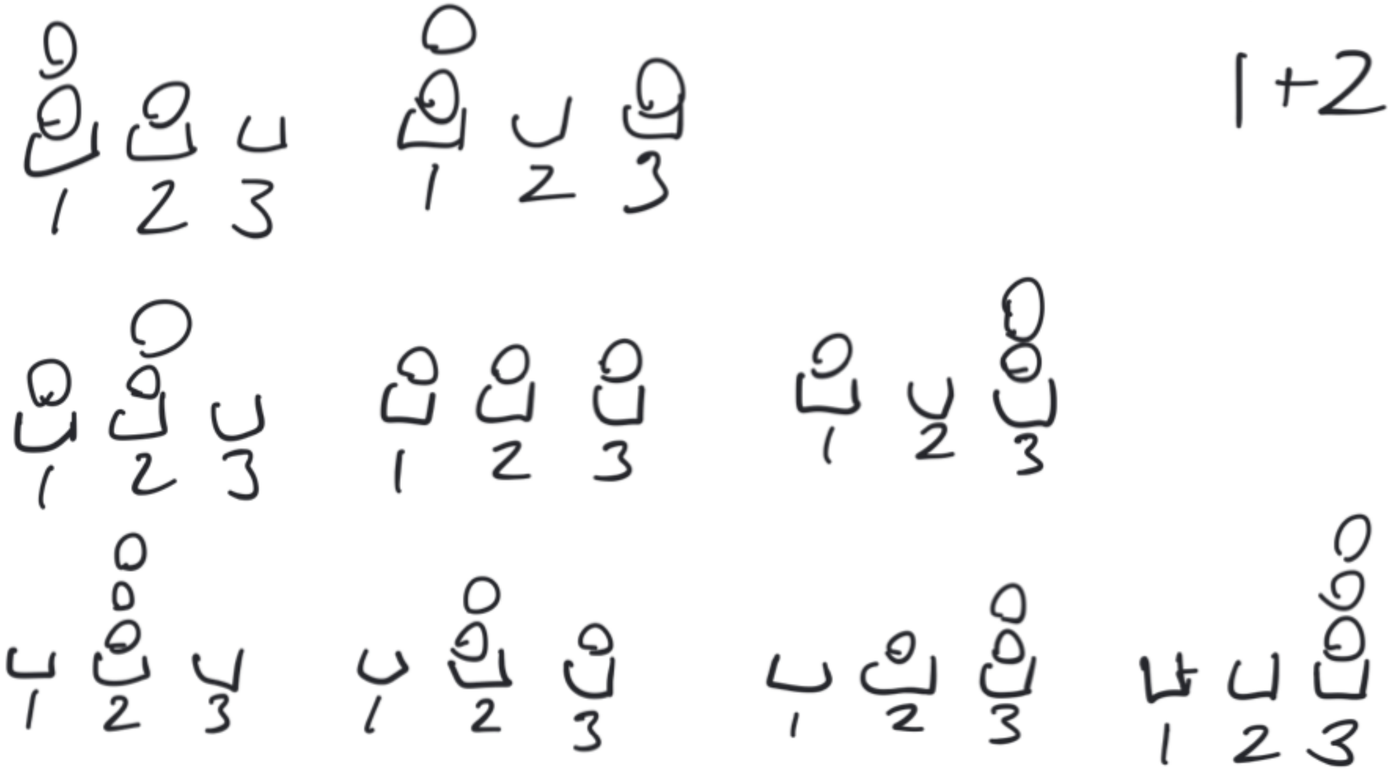
Unlabeled Balls in Labeled Bins

Q: How many ways are there to put n unlabeled balls into k labeled bins?

Example: How many ways are there to put 3 unlabeled balls in 3 labeled bins?

total # of possibilities

1+2+3+4 = 10

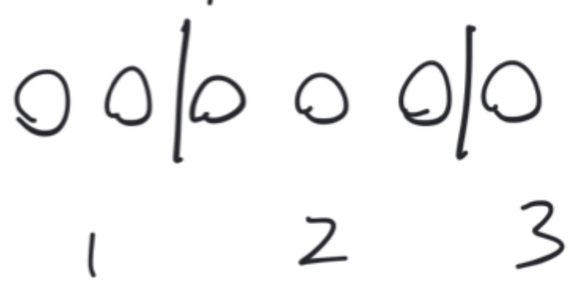


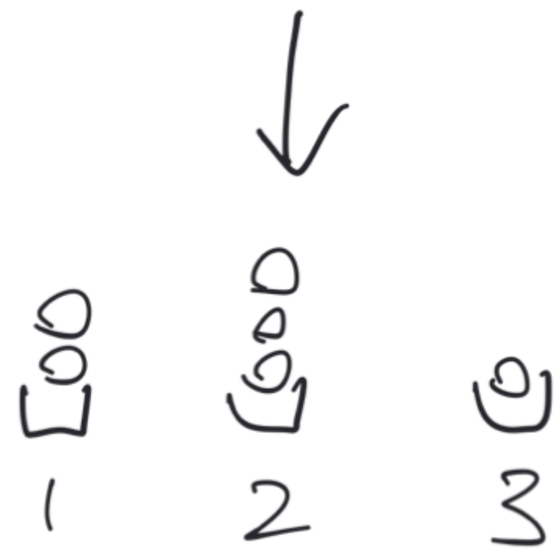
Unlabeled Balls in Labeled Bins

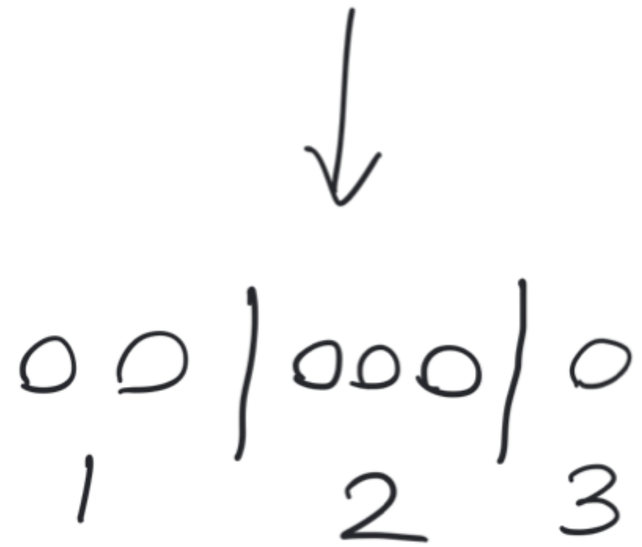
Q: How many ways are there to put n unlabeled balls into k labeled bins?

Trick: This is equivalent to placing k-1 dividing lines among the n balls.

Example: n=6, k=3







Unlabeled Balls in Labeled Bins

Q: How many ways are there to put n unlabeled balls into k labeled bins?

Trick: This is equivalent to placing k-1 dividing lines among the n balls.

Q: How many ways are there to do this?

Answer: $\binom{n+k-1}{k-1}$

Reason: There's a total of n+k-1 objects (n balls & k-1 bars). We need to choose which k-1 out of the n+k-1 objects are bars.

Example 1: If $n=3$ and $k=3$, this is $\binom{5}{2}=\frac{5\cdot\cancel{4}^{2}}{\cancel{2}\cdot 1}=10$

Example 2: If $n=5$ and $k=4$, this is $\binom{8}{3}=\frac{8\cdot 7\cdot\cancel{6}}{\cancel{3}\cdot\cancel{2}\cdot 1}=56$