

Pascal's Triangle

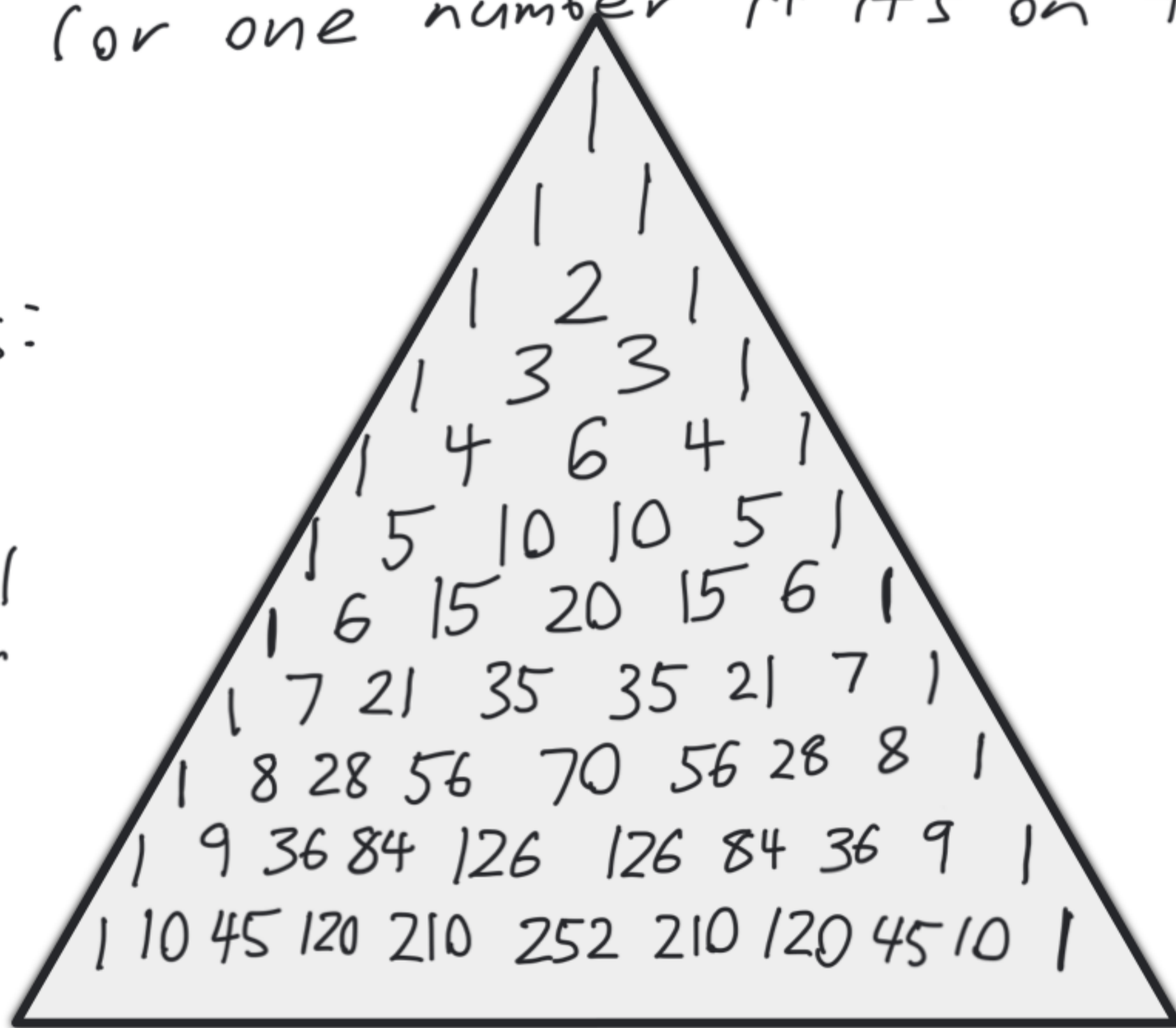
Pascal's Triangle: Start with a 1 at the top. Every other number is the sum of the two numbers above it (or one number if it's on the edge).

Examples:

$$3 = 1 + 2$$

$$56 = 35 + 21$$

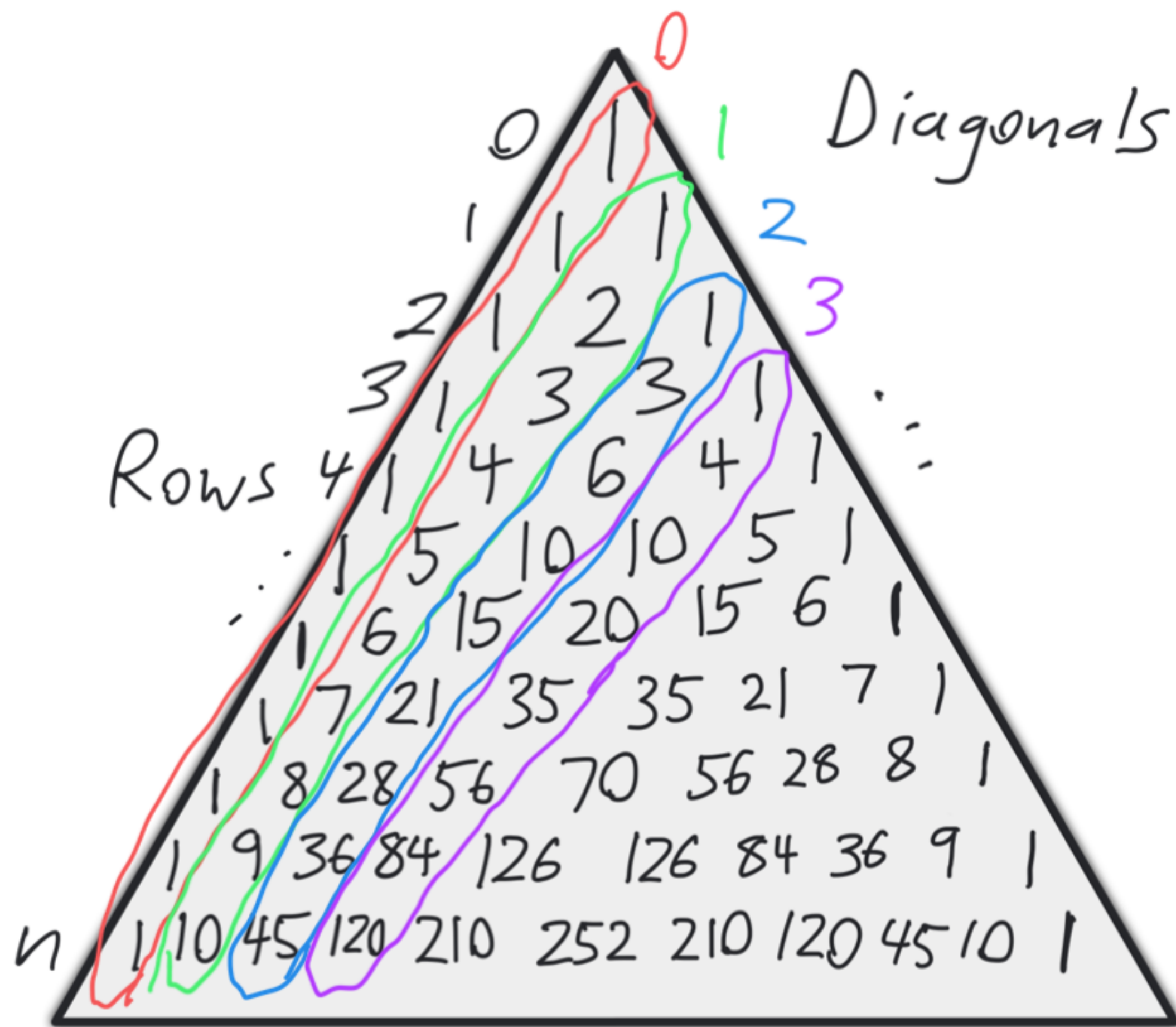
$$252 = 126 + 126$$



Q: What patterns can you see in Pascal's Triangle?

Pascal's Triangle Patterns

Diagonals of Pascal's Triangle:



0th diagonal: 1

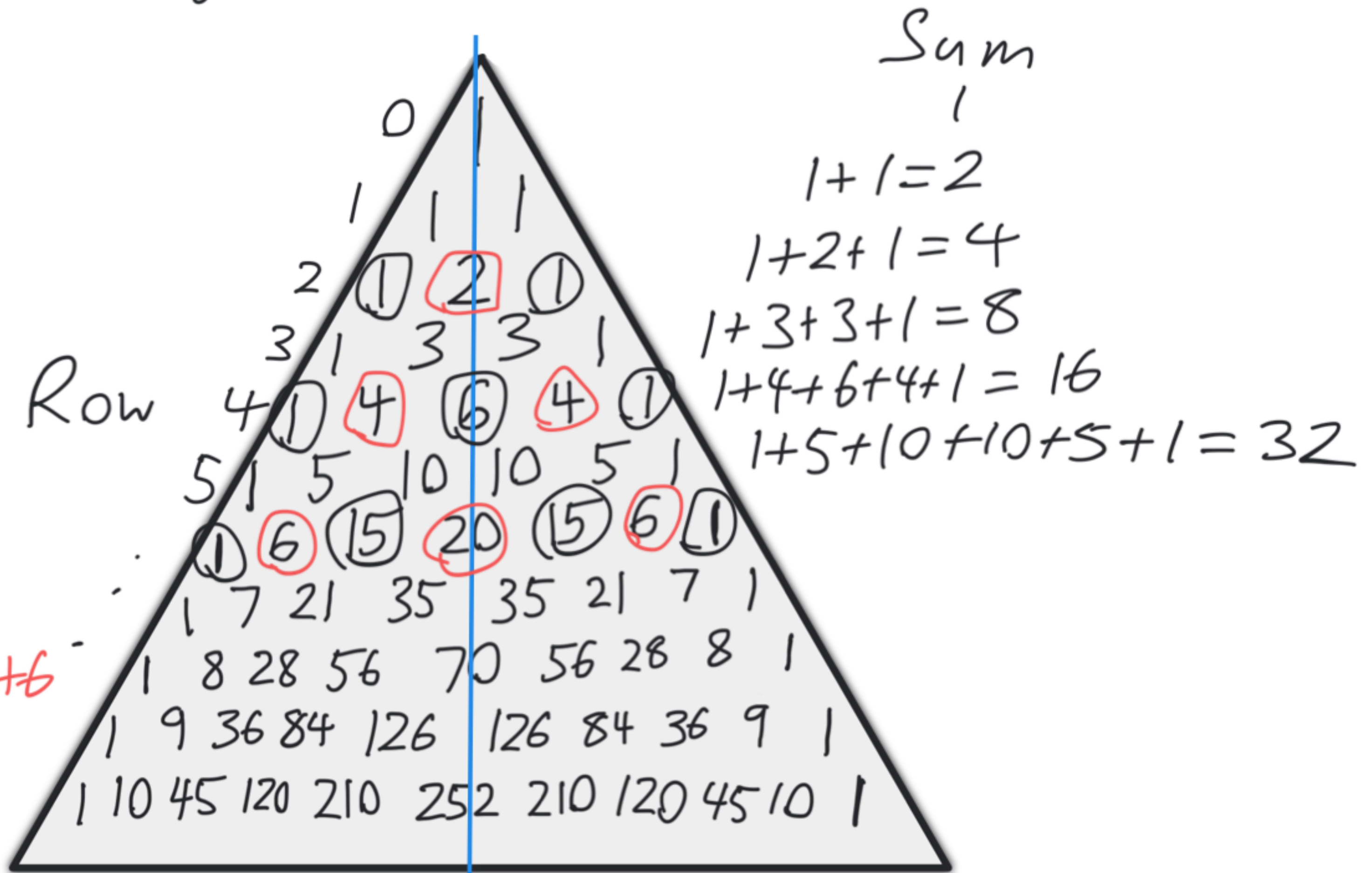
2nd diagonal:
Triangular numbers $\frac{n(n-1)}{2}$

1st diagonal: n

3rd diagonal: $\frac{n(n-1)(n-2)}{6}$
Example: $20 = \frac{6 \cdot 5 \cdot 4}{6}$

Pascal's Triangle Patterns

Pascal's Triangle is symmetric.



Examples:

$$1+1=2$$

$$1+6+1=4+4$$

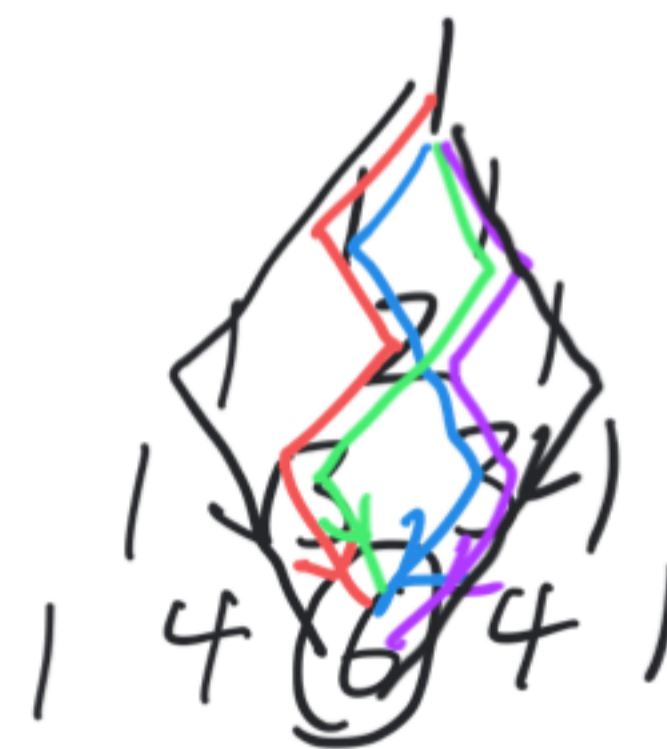
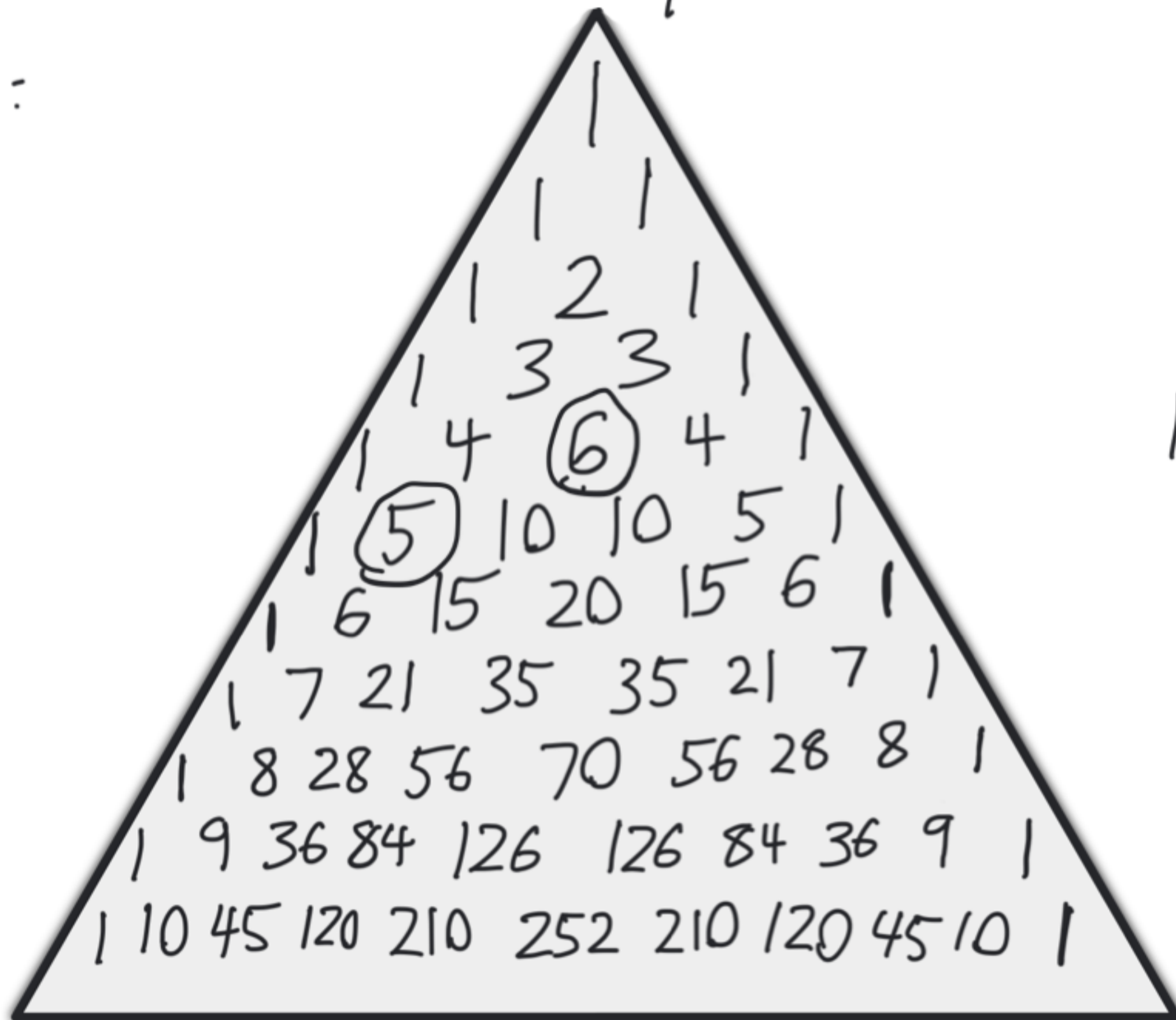
$$1+15+15+1=6+20+6$$

The sum of the numbers in row n is 2^n . Moreover, for all $n > 0$, the sum of the numbers in row n in even diagonals = the sum of the numbers in row n in odd diagonals.

Pascal's Triangle Patterns

Pattern: Each number gives the # of paths from the top to that number.

Example 1:



Pascal's Triangle Patterns

Hockey stick pattern: If you form a hockey stick by going down and to the left along a diagonal and then taking one step down and to the right, the sum of the numbers in the long part of the stick equals the number at the end of the stick.

Examples:

$$1 + 2 + 3 + 4 + 5 = 15$$

$$1 + 4 + 10 + 20 = 35$$

$$1 + 8 + 36 = 45$$

