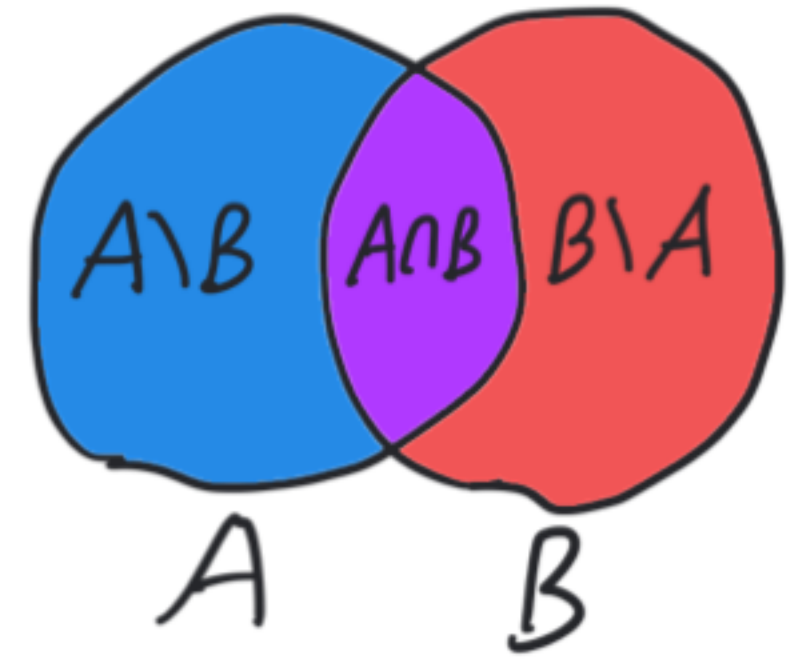
Inclusion/Exclusion with Two Sets



Inclusion/Exclusion: $|A \cup B| = |A| + |B| - |A \cap B|$

Proof:

$A = (A \setminus B) \cup (A \cap B)$, $B = (B \setminus A) \cup (A \cap B)$

$|A| = |A \setminus B| + |A \cap B|$, $|B| = |B \setminus A| + |A \cap B|$

$|A| + |B| - |A \cap B| = |A \setminus B| + |A \cap B| + |B \setminus A| + \cancel{|A \cap B|} - \cancel{|A \cap B|} = |A \cup B|$

If there are two sets of choices,

total # of choices = (# of choices in the first set) + (# of choices in the second set) − (# of choices which are in both sets)

Q: What is $|\{n \in [100] : 3 | n \vee 5 | n\}|$?

A: Divisible by 3: $|\{3a : a \in \mathbb{N} \wedge a \leq \frac{100}{3}\}| = \lfloor \frac{100}{3} \rfloor = 33$ numbers.

Divisible by 5: $\lfloor \frac{100}{5} \rfloor = 20$ numbers

Divisible by both 3 and 5 $\Leftrightarrow$ Divisible by 15: $\lfloor \frac{100}{15} \rfloor = 6$ numbers

$33 + 20 - 6 = \boxed{47}$